III. Solution by H. C. WILKES, Skull Run, West Virginia; and A. H. BELL, Hillsborough, Illinois.

Factoring, etc., $(x^2+2m)(x^2+3m)=m^2-\dfrac{n}{x}$. Let $x=n$; then $x=n$, $\pm\sqrt{-m-1}$, $\pm\sqrt{1-2m}$, which will be the five roots.

Or $(x^2+3m)(x^2+2m)=(m+\sqrt{\dfrac{n}{x}})(m-\sqrt{\dfrac{n}{x}}$. Assuming $x^3+3m=\sqrt{\dfrac{n}{x}}$, $x^3+2m=m-\sqrt{\dfrac{n}{x}}$. $x=\sqrt{-\dfrac{3m}{2}}$, $m=2\sqrt{\dfrac{n}{x}}$; hence $x=\dfrac{4n}{m^2}$. Substituting $m=2\sqrt{\dfrac{n}{x}}$ for $m$ in eq. 1, $x^5+10x^3\sqrt{\dfrac{n}{x}}+21n=0$. This can be developed, $x^{10}-58nx^5+441=0$. $\therefore x=\sqrt[5]{49n}$ or $\sqrt[5]{9n}$.

[The above is not strictly a solution, but affords a method of discovering integer roots, if any. The solu- of Professor Zerr is especially full and neat. EDITOR.]

Also solved by *F. P. MATZ*.

52. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

In how many ways can we arrange 12 friends of the MONTHLY, around a table, so that; (1) the editors may never be together, (2) Matz and Halsted may never be apart, and (2) Zerr and Ellwood may always have Gruber betwixt them?

Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

I. Considering one editor in position the other may occupy 9 places; but the first editor may take 12 places, and therefore the two take 108 positions. For each of these places the remaining nine mathematicians may be seated in $\lfloor 9$ ways, making $108\lfloor 9$ ways altogether.

II. If Matz and Halsted are never apart we may consider them as an element to be arranged as *each* of the other individuals. We then have $\lfloor 11$ ways of arranging them without regarding the *internal* arrangement of the group; this may be arranged in two ways. We, therefore, have $2\lfloor 11$ as the number of arrangements.

III. By the same reasoning as in the last case we have the number of arrangements$=2\lfloor 10$.

NOTE.—No solution of problem 53 has as yet been received. The published solution of problem 49, in last issue, should have been credited to Prof. J. H. Grove, Howard Payne College, Brownwood, Texas.

## PROBLEMS.

59. Proposed by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

Demonstrate the identity $2^{2n+1}\dfrac{d^n}{dx^n}\left(x^{n+\frac{1}{2}}\dfrac{d^{n+1}}{dx^{n+1}}e^{\sqrt{x}}\right)=e^{\sqrt{x}}$.

60. **Proposed by Professor C. E. WHITE, Trafalgar, Indiana.**

Prove that every algebraic equation can be transformed into another equation of the same degree, but which wants its $n^{th}$ term.

61. **Proposed by J. A. CALDERHEAD, A. B., Superintendent of Schools, Limaville, Ohio.**

Given $x^2+x\sqrt{xy}=10$, and $y^2+y\sqrt{xy}=20$ to find $x$ and $y$ by quadratics.

# CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

## SOLUTIONS OF PROBLEMS.

40. **Proposed by F. P. MATZ, D. Sc., Ph.D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.**

The closed portion of the curve known as "The Cocked Hat," equation

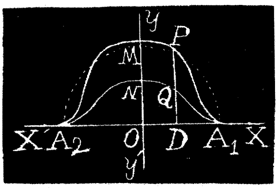
$$x^4+x^2y^2+4ax^2y-2a^2x^2+3a^2y^2-4a^3y+a^4=0,$$

revolves around the axis of $y$. Find the *campanulate* volume generated. If the same portion of the curve revolve around the axis of $x$, find the *fusiform* volume generated. Also, determine the area of this closed portion of the curve.

**Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Taxas; W. C. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts; and the PROPOSER.**

Solving the equation for $x^2$ we get $x^2=\pm\frac{1}{2}y\sqrt{y^2+8ay}-\frac{1}{2}(y^2+4ay-2a^2)$.

$\therefore$ The campanulate volume generated by the area $MPA_1QNM$ is




$$V=\frac{\pi}{2}\int_0^a(y\sqrt{y^2+8ay}-y^2-4ay+2a^2)dy+$$

$$\frac{\pi}{2}\int_0^{\frac{1}{3}a}(y\sqrt{y^2+8ay}+y^2+4ay-2a^2)dy.$$

$$=\frac{\pi}{2}\Big[\tfrac{1}{3}(y^2+8ay)^{\frac{3}{2}}-2a(y-4a)\sqrt{y+8ay}+32a^3\log$$

$$\Big\{y+4a+\sqrt{y^2+8ay}\Big\}-\tfrac{1}{3}y^3-2ay^2+2a^2y\Big]_0^a+\frac{\pi}{2}\Big[\tfrac{1}{3}y^3+2ay^2-2a^2y+\tfrac{1}{3}(y^2+$$

$$8ay)^{\frac{3}{2}}-2a(y+4a)\sqrt{y^2+8ay}+32a^3\log\Big\{y+4a+\sqrt{y^2+8ay}\Big\}\Big]_0^{\frac{1}{3}a}$$

$$=\tfrac{4}{3}\pi a^3(12\log3-13).$$

[ZERR, MATZ, and BLACK.]